*E4732 CaseStudy 1 name: Yunfei Yan UNI: yy2516*

*(1) Description of coding environment.*

*I use Mobaxterm to code this case study 1 @clic-lab.cs.columbia.edu in linux environment, my compiler is git and my editor is vim.*

*I haven’t used any external library to do the math calculation.*

*(2) My answers to the question.*

***Notice: all the results with bold and underline font are the ones same as BS formula.***

*FFT:*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *eta = 0.25* | *N = 2 ^ 8* | *N = 2 ^ 10* | *N = 2 ^ 12* | *N = 2 ^ 14* |
| *Alpha = -2* | *K = 900; European put option price =* ***0.044996***  *K = 1100; European put option price =* ***1.148194***  *K = 1300; European put option price =* ***9.588557***  *K = 1500; European put option price =* ***40.256689*** | | | |
| *Alpha = -5* |
| *Alpha = -10* |
| *Alpha = -20* |

*FRFFT:*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *K = 1500* | *N = 2 ^ 7* | *N = 2 ^ 8* | *N = 2 ^ 9* | *N = 2 ^ 10* |
| *Alpha = -2* | *40.256137* | ***40.256689*** | ***40.256689*** | ***40.256689*** |
| *Alpha = -5* | *40.257111* | ***40.256689*** | ***40.256689*** | ***40.256689*** |
| *Alpha = -10* | *40.256274* | ***40.256689*** | ***40.256689*** | ***40.256689*** |
| *Alpha = -20* | *40.291469* | ***40.256689*** | ***40.256689*** | ***40.256689*** |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *K = 1300* | *N = 2 ^ 7* | *N = 2 ^ 8* | *N = 2 ^ 9* | *N = 2 ^ 10* |
| *Alpha = -2* | *9.588917* | ***9.588557*** | ***9.588557*** | ***9.588557*** |
| *Alpha = -5* | *9.588330* | ***9.588557*** | ***9.588557*** | ***9.588557*** |
| *Alpha = -10* | *9.588567* | ***9.588557*** | ***9.588557*** | ***9.588557*** |
| *Alpha = -20* | *9.587092* | ***9.588557*** | ***9.588557*** | ***9.588557*** |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *K = 1100* | *N = 2 ^ 7* | *N = 2 ^ 8* | *N = 2 ^ 9* | *N = 2 ^ 10* |
| *Alpha = -2* | *1.147974,* | ***1.148194*** | ***1.148194*** | ***1.148194*** |
| *Alpha = -5* | *1.148286* | ***1.148194*** | ***1.148194*** | ***1.148194*** |
| *Alpha = -10* | *1.148196* | ***1.148194*** | ***1.148194*** | ***1.148194*** |
| *Alpha = -20* | *1.148242* | ***1.148194*** | ***1.148194*** | ***1.148194*** |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *K = 900* | *N = 2 ^ 7* | *N = 2 ^ 8* | *N = 2 ^ 9* | *N = 2 ^ 10* |
| *Alpha = -2* | *0.045145* | ***0.044996*** | ***0.044996*** | ***0.044996*** |
| *Alpha = -5* | *0.044967* | ***0.044996*** | ***0.044996*** | ***0.044996*** |
| *Alpha = -10* | *0.044998* | ***0.044996*** | ***0.044996*** | ***0.044996*** |
| *Alpha = -20* | *0.044994* | ***0.044996*** | ***0.044996*** | ***0.044996*** |

*COS:*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *N = 2 ^ 9* | *[ a, b ] = [ -2, 2 ]* | *[ a, b ] = [ -5, 5 ]* | *[ a, b ] = [ -10, 10 ]* | *[ a, b ] = [ -20, 20 ]* |
| *K = 900* | ***0.044996*** | ***0.044996*** | ***0.044996*** | ***0.044996*** |
| *K = 1100* | ***1.148194*** | ***1.148194*** | ***1.148194*** | ***1.148194*** |
| *K = 1300* | ***9.588557*** | ***9.588557*** | ***9.588557*** | ***9.588557*** |
| *K = 1500* | ***40.256689*** | ***40.256689*** | ***40.256689*** | ***40.256689*** |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *N = 2 ^ 8* | *[ a, b ] = [ -2, 2 ]* | *[ a, b ] = [ -5, 5 ]* | *[ a, b ] = [ -10, 10 ]* | *[ a, b ] = [ -20, 20 ]* |
| *K = 900* | ***0.044996*** | ***0.044996*** | ***0.044996*** | *0.045021* |
| *K = 1100* | ***1.148194*** | ***1.148194*** | ***1.148194*** | *1.148103* |
| *K = 1300* | ***9.588557*** | ***9.588557*** | ***9.588557*** | *9.588671* |
| *K = 1500* | ***40.256689*** | ***40.256689*** | ***40.256689*** | *40.256553* |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *N = 2 ^ 7* | *[ a, b ] = [ -2, 2 ]* | *[ a, b ] = [ -5, 5 ]* | *[ a, b ] = [ -10, 10 ]* | *[ a, b ] = [ -20, 20 ]* |
| *K = 900* | ***0.044996*** | ***0.044996*** | *0.045026* | *0.224234* |
| *K = 1100* | ***1.148194*** | ***1.148194*** | *1.148095* | *1.318946* |
| *K = 1300* | ***9.588557*** | ***9.588557*** | *9.588680* | *8.995516* |
| *K = 1500* | ***40.256689*** | ***40.256689*** | *40.256542* | *40.215988* |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *N = 2 ^ 6* | *[ a, b ] = [ -2, 2 ]* | *[ a, b ] = [ -5, 5 ]* | *[ a, b ] = [ -10, 10 ]* | *[ a, b ] = [ -20, 20 ]* |
| *K = 900* | ***0.044996*** | *0.045038* | *0.246895* | *-4.998904* |
| *K = 1100* | ***1.148194*** | *1.148078* | *1.315780* | *-8.704494* |
| *K = 1300* | ***9.588557*** | *9.588698* | *8.961801* | *7.075620* |
| *K = 1500* | ***40.256689*** | *40.256519* | *40.218453* | *51.965224* |

*According to the formula: c1 = ( r – q ) T; c2 = sigma ^ 2 \* T; c4 = 0; L = 10, then we have the best choice for [ a, b ] is [ -8.9 \* 0.001 – 2.12, -8.9 \* 0.001 + 2.12 ] approximately equal to [ -2, 2 ].*

*(3 ) Compare and conclusion.*

*a. Theoretically, the speed about the 3 methods should be ranked like this: COS >FRFFT>FFT. Due to the machine I used is the one in Columbia Clic-lab, which is very powerful, I didn’t feel the big speed difference between these methods.*

*b. COS estimation: [ a, b ], N*

*FFT estimation: alpha, eta, N*

*FRFFT estimation: alpha, eta, N, lambda*

*c. They can only be used in the path-independent vanilla options, and when the option is highly out of the money, they are useless.*

*d. Both FFT and FRFFT could give us N option prices at N different strikes. For example:*

*fft K*

*0.000050, 653.861889*

*0.000502, 721.311383*

*0.004166, 795.718668*

*0.028460, 877.801479*

*0.160517, 968.351588*

*0.750762, 1068.242445*

*2.928012, 1178.437601*

*9.588557, 1300.000000*

*26.598111, 1434.102237*

*63.186041, 1582.037867*

*130.285553, 1745.233881*

*236.923841, 1925.264472*

*386.921106, 2123.866220*

*578.497978, 2342.954845*

*806.965025, 2584.643681*

*The pity is that not all the price can be used ( eg: deep out of the money ). The COS method give us only one price each time.*

*e. These three methods are model free, all they need is a simple characteristic function. Also, the COS separate the model from its payoff, which make it more useful.*

*f. According to the results, the FFT method is not sensitive to the alpha and N, which means it is comparative stable.*

*g. The FRFFT are a little sensitive to the alpha and N, it seems that no matter what alpha we use, N = 2 ^ 7 is always not a good choice for us. I think we should use N >= 2 ^ 8 in practical to price the option.*

*h. The COS is very sensitive to the coefficient we selected. We can see that the interval and N are somewhat complementary. With the smaller N, the interval should be narrower to fit for the N’s decrease.*

*(4) Description of the logic present in the written source code*

*I am a new coder and have no experience before. So I used the simplest method to calculate the result in the main function step by step. There is no other function except main in all my 3 codes. Every time I change the coefficients manually to get the result.*

*I download my code from Mobaxterm directly with .cc type. You can open it using notepad ( I have already tried, and it works. ) If you cannot open it, please contact me. Thank you!*